Spanning eulerian subdigraphs avoiding k prescribed arcs in semicomplete digraphs

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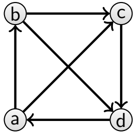
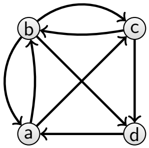
Joint work with: Jørgen Bang-Jensen and Hugues Déprés (and F. Havet)

Definitions



A tournament
(and a semicomplete digraph)



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(not a tournament)

A digraph is **eulerian** if it is connected and every vertex has its in-degree equal to its out-degree.

A digraph containing a spanning eulerian subdigraph is called **supereulerian**.

If a digraph has a Hamilton cycle, then it is supereulerian.

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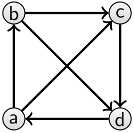

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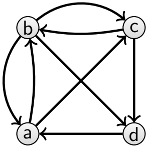

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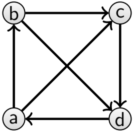

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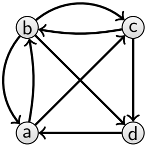

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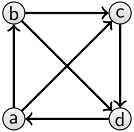

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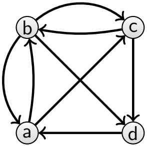

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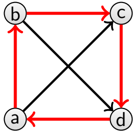

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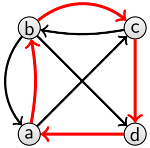

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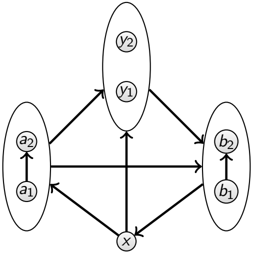

Contains a spanning eulerian subdigraph, but no Hamilton cycle

Why does the above contain no Hamilton cycle?

Because any path from y_1 to y_2 contains x and any path from y_2 to y_1 also contains x.

Why is it supereulerian?

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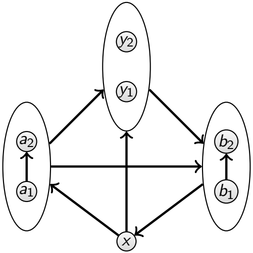

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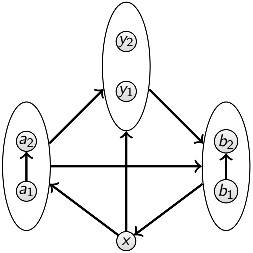

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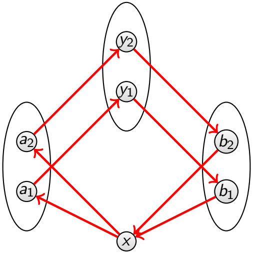

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A **Eulerian factor**, F, is a subdigraph where $d_F^+(x) = d_F^-(x) \geq 1$ for all vertices x.

Each connected component of a Eulerian factor is called a **component**.

A spanning eulerian subdigraph is a factor with one component.

A non-supereulerian digraph

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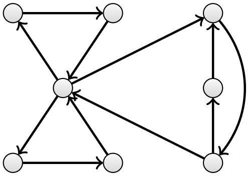

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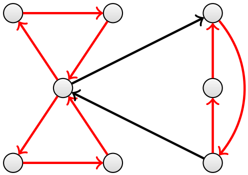

A eulerian factor in
a non-supereulerian digraph

Known results

Theorem 1 (Camion): Every strongly connected semicomplete digraph has a hamiltonian cycle.

Bang-Jensen and Thomassé made the following conjecture in 2011 which may be seen as a generalization of Camion's theorem ($\lambda(D)$ is the arc-connectivity of D and $\alpha(D)$ is the independence number of D).

Conjecture 2 (Bang-Jensen and Thomassé): Every digraph D with $\lambda(D) \geq \alpha(D)$ is supereulerian.

This is still open, even for $\alpha(D) = 2$.

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Conjecture 4 (Bang-Jensen, Havet and AY): Let $D = (V, A)$ be a $(k+1)$-arc-strong semicomplete digraph and let $A' \subset A$ be any set of k arcs of D. Then $D \setminus A'$ has is supereulerian.

If we look at vertex-connectivity instead of arc-connectivity, then ...

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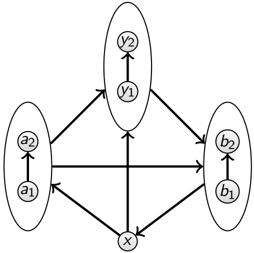

D is 2-arc-connected and 1-strong.

$D \setminus y_1y_2$ has no Hamilton cycle, so the Fraise-Thomassen result cannot be extended to arc-connectivity.

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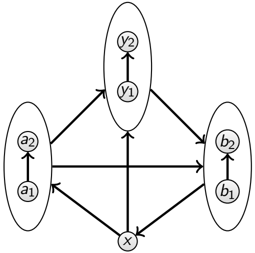

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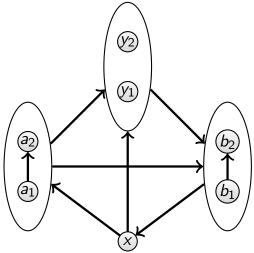

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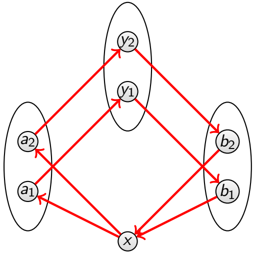

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So we know there is a eulerian factor in $D \setminus A'$. We will now use this to prove the following result.

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Outline of proof of Theorem 7

Let D be a $(2k+1)$-arc-strong semicomplete digraph, $A' \subset A$ with $|A'| \leq k$ and $D' = D \setminus A'$.

By Theorem 6, let $F = C_1 \cup C_2 \cup \cdots \cup C_p$ be a eulerian factor in D' ($p \geq 2$ otherwise we are done).

As D is $(2k+1)$-arc-strong, $|V(D)| > 2k+1$ and there exists a vertex u adjacent to all other vertices in D'.

u is called **universal**.

Without loss of generality assume $u \in V(C_1)$ and $|V(C_p)|$ is maximum.

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Let L_i be an in-branching in C_i for all $i = 1, 2, \ldots, p-1$, such that u is the root of L_1.

Let $L = L_1 \cup L_2 \cup \cdots \cup L_{p-1}$. Note that $V(L) = V(D') \setminus V(C_p)$.

A S-path is a sequence of vertices $p_0 p_1 p_2 \ldots p_l$, such that the following holds.

- $p_i \in V(L)$ when $0 \leq i < l$ and $p_l \in V(C_p)$.
- $p_i p_{i+1}$ is an $(F - L)$-arc (i.e. $p_i p_{i+1} \in A(F) \setminus A(L)$) or $p_{i+1} p_i$ is a non-F-arc (i.e $p_{i+1} p_i \in A(D') \setminus A(F)$).

Let $X \subseteq V(L)$ contain all non-start-vertices of S-paths.

Let $Y \subseteq V(L)$ contain all start-vertices of S-paths.

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A S-path is a sequence of vertices $p_0 p_1 p_2 \ldots p_l$, such that the following holds.

- $p_i \in V(L)$ when $0 \leq i < l$ and $p_l \in V(C_p)$.
- $p_i p_{i+1}$ is an $(F - L)$-arc (i.e. $p_i p_{i+1} \in A(F) \setminus A(L)$) or $p_{i+1} p_i$ is a non-F-arc (i.e $p_{i+1} p_i \in A(D') \setminus A(F)$).

Let $X \subseteq V(L)$ contain all non-start-vertices of S-paths.

Let $Y \subseteq V(L)$ contain all start-vertices of S-paths.

Outline of proof of Theorem 7

Without loss of generality u dominates C_p (we will show this on the board).

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1. If $y \in Y$, then y cannot dominate C_p.

2. So $u \notin Y$, implying that $u \in X$ and $X \neq \emptyset$.

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4. There are at least $k + 1$ (X, Y)-arcs belonging to F.

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Conclusion

Using a similar approach, the bound of $2k+1$ can be improved.

Theorem 8 (Bang-Jensen, Déprés and AY): Let $D = (V, A)$ be a $\lceil \frac{6k+1}{5} \rceil$-arc-strong semicomplete digraph and let $A' \subset A$ be any set of k arcs of D. Then $D \setminus A'$ has is supereulerian.

However the proof of this is quite technical, even though it uses similar techniques to the above.

This appraoch could potentially be used to improve the $\lceil \frac{6k+1}{5} \rceil$ bound slightly, but it doesn't seem like this approach can be used to give the $k+1$ bound conjectured to be true.

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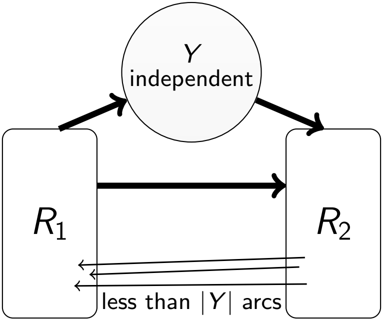
The end

Thank you

Appendix

Theorem (Bang-Jensen, Havet and AY): If D is a digraph then D has no eulerian factor if and only if $V(D)$ can be partitioned into R_1, R_2 and Y such that the following holds.

- Y is independent.
- $d(R_2, Y) = 0$, $d(Y, R_1) = 0$ and $d(R_2, R_1) < |Y|$.



There are no arcs from R_2 to Y and no arcs from Y to R_1 and less than $|Y|$ arcs from R_2 to R_1.