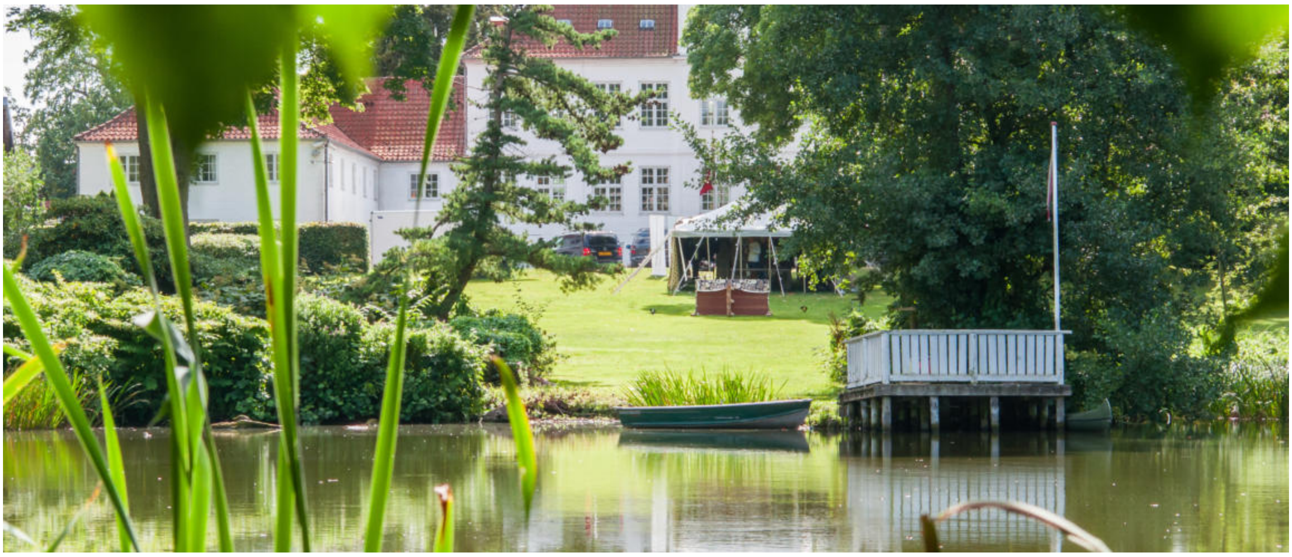
13TH NORDIC COMBINATORIAL CONFERENCE

Schæffergården, Copenhagen, Denmark

August 5-7, 2019



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Welcome to NORCOM 2019!

The Nordic Combinatorial Conferences are held every third year – the venue and organisation rotating among the Nordic countries. At NORCOM, mathematicians from the Nordic countries as well as from other countries meet and interact in an informal, relaxed atmosphere. All fields of mathematics related to combinatorics and discrete mathematics are welcome.

About the Nordic Combinatorial Conferences

This is the 13th Nordic Combinatorial Conference. Previous conferences have taken place in Denmark, Finland, Iceland, Norway and Sweden. To see a list of all former conferences and learn more, see the NORCOM homepage http://www.nordiccombinatorics.org/ .

This year's organisers

- Lars Døvling Andersen, Aalborg University
- Peter Beelen, Technical University of Denmark
- Trygve Johnsen, UiT - The Arctic University of Norway
- Bjarne Toft, University of Southern Denmark

NORCOM steering committee

- Einar Steingrímsson, University of Strathclyde
- Lars Døvling Andersen, Aalborg University
- Patric Östergård, Aalto University
- Petter Brändén, KTH Royal Institute of Technology
- Trygve Johnsen, UiT - The Arctic University of Norway

Forthcoming conferences

The 14th NORCOM conference will take place in 2022 in Norway. Stay updated at http://www.nordiccombinatorics.org/future.html

Plenary speakers (in order of appearance):

- Jack Edmonds
- Kathie Cameron, Wilfrid Laurier University
- Peter Michaelsen, Hvidebæk Pastorat
- Olav Geil, Aalborg University

Speakers (alphabetically by surname):

- Jørgen Bang-Jensen, University of Southern Denmark
- Ángela Barbero, Universidad de Valladolid and Øyvind Ytrehus, Simula UiB
- Ignacio Cascudo Pueyo, Aalborg University
- Carl Johan Casselgren, Linköping University
- Mikhail Ganzhinov, Aalto University
- Oliver W. Gnilke, Aalborg University
- Jonas Granholm, Linköping University
- Jaron Skovsted Gundersen, Aalborg University
- Daniel Heinlein, Aalto University
- Anders Nedergaard Jensen, Aarhus University
- Trygve Johnsen, UiT - The Arctic University of Norway
- Leif Kjær Jørgensen, Aalborg University
- Dani Kotlar, Tel-Hai College
- William Paavola, Aalto University
- Liam Solus, KTH Royal Institute of Technology
- Bjarne Toft, Southern University of Denmark
- Anders Yeo, University of Southern Denmark
- Patric Östergård, Aalto University

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Programme:

Monday, August 5

12.00 - 13.00 Lunch

13.00 - 13.30 Welcome and introduction to Schæffergården by a representative from the Foundation for Danish-Norwegian Cooperation (which owns Schæffergården and has supported the conference considerably)

13.30 - 14.30 Jack Edmonds: Origins of NP and P. The full story. NP ≠ P.

14.30 - 15.00 Coffee and tea

15.00 - 17.30 Contributed talks (chair Trygve Johnsen)

- Anders Yeo: ***Spanning eulerian subdigraphs avoiding k prescribed arcs in tournaments***
- Carl Johan Casselgren: ***Latin squares and Latin cubes with forbidden entries***
- Dani Kotlar: ***On sequential basis replacement in matroids***
- Leif Kjær Jørgensen: ***Automorphisms of an srg(162,21,0,3)***
- Patric Östergård: ***r-sparse Steiner Triple Systems***

18.00 Dinner

Tuesday, August 6

8.00 - 9.00 Breakfast

9.00 - 10.00 Kathie Cameron: Parity theorems about trees and cycles

10.00 - 10.30 Coffee and tea

10.30 - 12.00 Contributed talks (chair Carl Johan Casselgren)

- Jonas Granholm ***A localization method in Hamiltonian graph theory***
- Jørgen Bang-Jensen**:** ***Good acyclic orientations, antistrong digraphs and matroids***
- Mikhail Ganzhinov: ***Construction of biangular line sets in low-dimensional Euclidian spaces***

12.00 - 13.00 Lunch

13.00 - 13.30 Peter Michaelsen: The Nordic running-fight games Daldøs and Sáhkku

13.30 - 15.00 Contributed talks (chair Bjarne Toft)

- Bjarne Toft: ***Hex: the full story***
- Ignacio Cascudo Pueyo: ***Squares of cyclic codes***
- Oliver W. Gnilke: ***Symmetric mosaics***

15.00 - 15.30 Coffee and tea

15.30 - 17.30 Contributed talks (chair Leif Kjær Jørgensen)

- Daniel Heinlein: ***Degree tables***
- Jaron Skovsted Gundersen: ***Improved bounds on the threshold gap in ramp secret sharing***
- Liam Solus: ***Real zeros and the alternatingly increasing property in algebraic combinatorics***
- William Paavola: ***Quaternary complex Hadamard matrices of order 18***

18.00 Conference dinner

Wednesday, August 7

8.00 - 9.00 Breakfast

9.00 - 10.00 Olav Geil: *Bounding the number of affine roots – with applications in communication theory*

10.00 - 10.30 Coffee and tea

10.30 - 12.00 Contributed talks (chair Patric Östergård)

- Anders Nedergaard Jensen: ***Tropical homotopy continuation***
- Ángela Barbero & Øyvind Ytrehus: ***Bounds for convolutional codes***
- Trygve Johnsen: ***Generalized weights of rank metric codes, a combinatorial approach***

12.00 Closing and lunch

Abstracts main speakers (in order of appearance)

Origins of NP and P. The Full Story. NP≠P.

Jack Edmonds

Inspired by the new book, 'Hex. The Full Story', I present an account beginning for me in 1960 of NP ∩ coNP and the conjecture NP≠P. Prompted by the Konig-Egervary polyhedral theory of 1931 for the optimum assignment problem, the Baruvka algorithm of 1926 for minimum spanning trees, and the Dantzig-Fulkerson-Johnson polyhedral algorithm of 1954 for the TSP, I spent several years trying to find a polynomial time algorithm for TSP.

Failing that, I conjectured NP≠P in 1966. While looking, I did find polynomial time polyhedral algorithms for some other problems, including OBS: Given a digraph G, specified root nodes r(i), and a cost for each edge of G, find a least cost collectionof edge-disjoint directed spanning trees in G, rooted respectively at the nodes r(i), i.e., r(i)-branchings in G.

The extensively treated min cost network flow problem is a special case of OBS. However OBS does not reduce to it. Curiously, the simplest way which I know to describe a good algorithm for OBS is by matroid intersection for general matroids.

Prompted by Julius Petersen's theorem of 1891, I did find a polynomial time algorithm for finding, in a graph G with weighted edges, a min cost total weight, spanning, degree 2, subgraph G' of G. However G' is perhaps not connected.

This talk is related to talks given in Lisbon on Feb 20 (Técnico Lisboa), and at the Sorbonne in Paris on May 3, and at a Celebration of the work of Stephen Cook in 50 Years of Computational Complexity, Fields Institute, Toronto, on May 9, 2019: "God provides only a few glimpses of heaven".

Monday 13.30

Parity Theorems About Trees and Cycles

Kathie Cameron
Wilfrid Laurier University
Waterloo, Canada

The famous Smith Theorem (Tutte 1946) says that if G is a 3-regular graph and e is an edge of G, then the number of hamiltonian cycles of G containing e is even.

An obvious corollary of any theorem which says that the number of objects is even is: Given one of the objects, there exists another. Given one hamiltonian cycle containing e, Tutte's elegant proof does not provide an algorithm for finding another.

Andrew Thomason (1978) extended Smith's Theorem to any graph where all vertices have odd degree.

Andrew Thomason's proof constructs a graph $X(G)$ which he calls a lollipop graph such that, when G is simple, the odd-degree vertices of $X(G)$ correspond precisely to the hamiltonian cycles of G containing e. This provides an algorithm for finding a second hamiltonian cycle containing e by walking in $X(G)$ from a given odd-degree vertex to another odd-degree vertex. Unfortunately, this elegant algorithm is exponential, even for 3-regular graphs. Jack Edmonds and I use the term *exchange graph* for a graph like Thomason's in which the odd-degree vertices correspond to the objects of interest. We have found that many theorems which say that the number of objects is even can be proved by constructing an exchange graph.

A graph is called *eulerian* if every vertex has even degree. Shunichi Toida (1973) proved that if G is eulerian and e is an edge of G, then the number of cycles of G containing e is odd.

Thomason's and Toida's Theorems say that for a given edge e in a graph G, the number of cycles containing e and all the odd-degree vertices is even if all vertices of G have odd degree and is odd if all vertices of G have even degree. Carsten Thomassen and I showed that the parity of the number of cycles containing e and all the odd-degree vertices is even as soon as G has an odd-degree vertex: If G is a graph and e is an edge of G, then the number of cycles of G containing e and all the odd-degree vertices is odd if and only if G is eulerian. Our proof is not algorithmic. I will give an exchange graph proof of this theorem and an extension of it to trees.

Tuesday 9.00

The Nordic running-fight games Daldøs and Sáhkku

Peter Michaelsen, Hvidebæk Pastorat

In his book *De Ludis Orientalibus*, Oxford 1694, the English orientalist Thomas Hyde (1636-1703) devoted a chapter: *Historia Tabiludii* to a category of board games, which was popular among Arabs, Turks and Persians in the Middle East at that time. One of these games, *tâb*, has been played in Egypt since at least the 13th c. A.D.

Following Hyde, "Tâb games", along with "running-fight" games, has been used to designate this class of games, which essentially combines the methods of race games (like backgammon and ludo) and the goal of elimination-based war games (such as draughts). Like in race games, pieces are moved along linear tracks based on the cast of dice or other lots; but like in draughts, the object is to capture opponent's pieces.

This family of dice board games has or had members in a lot of countries, including Greece, large parts of Africa, India, the Comoros Isles, and even as far away as among the Nivkh people in Eastern Siberia.

In this lecture I will give a short presentation of the two Scandinavian members of the "Tâb" or "running-fight" family, the Danish-Norwegian *daldøs*(*a*) and the Sámi *sáhkku*, and discuss the interesting question how these originally Arab games might have spread to Northern Europe.

My first article on Daldøs and Sáhkku was published in the Danish journal *Ord & Sag* 20 years ago. Two years later a large part of the journal *Board Games Studies*, issue 4, was devoted to articles about these and other "running-fight games", written by several researchers. Since then a lot of new information has been discovered, and in 2012 I tried to make a survey of the situation in my article "Un Jeu Médieval Arabe en Scandinavie?" in the magazine *Histoire et Images Medievales*, no.28.

During the last few years, archeologists have found several new traces of this type of games around in Europe. Besides, the almost extinct Sáhkku game has experienced a renaissance in many places in Norway, Northern Sweden and Northern Finland, thanks to the efforts of Sámi board game enthusiasts in Norway. They have already contributed with important new information about the rules, history and geographical distribution of Sáhkku and other traditional Sámi games, and will undoubtedly continue to do so in the future.

Tuesday 13.00

BOUNDING THE NUMBER OF AFFINE ROOTS

with applications in communication theory

Olav Geil
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NORCOM 2019
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We consider multivariate polynomials over arbitrary fields, with a special interest in finite fields. We study the number of affine roots – with or without multiplicity. The methods used mainly rely on simple Gröbner basis theory. Besides being of general mathematical interest the results have several applications/implications in areas such as cryptography and coding theory, but also in function field theory.

Wednesday 9.00

Abstracts other speakers (alphabetically by surname)

Good acyclic orientations, antistrong digraphs and matroids

Jørgen Bang-Jensen
University of Southern Denmark*

NORCOM 2019, August 6, 2019

An out-branching B_s^+ (in-branching B_s^-) rooted at the vertex s in a digraph $D = (V, A)$ is a spanning tree in the underlying graph of D with the property that every vertex $v \neq s$ has exactly one arc entering (leaving) it in B_s^+. Equivalently, an out-branching (in-branching) of $D = (V, A)$ is a spanning subdigraph of D on $|V| - 1$ arcs such that s can reach (be reached by) every other vertex using only arcs in the branching. Edmonds proved that a digraph has k arc-disjoint out-brachings, all rooted at s if and only if there are k arc-disjoint paths from s to every other vertex in D. There are several different polynomial algorithms for finding k arc-disjoint out-branchings in a digraph that satisfies the condition above. One is based on matroid techniques and another on flow calculations.

Thomassen proved that it is NP-complete to decide whether a digraph has an out-branching B_s^+ and an in-branching B_s^- that are arc-disjoint. For acyclic digraphs such a pair can only exist if s (t) is the unique vertex of in-degree (out-degree) zero and Bang-Jensen, Thomassé and Yeo showed that the problem is polynomial for acyclic digraphs.

In this talk we will first discuss the problem of deciding whether a given graph G has an acyclic orientation D with an out-branchings B_s^+ and an in-branching B_s^- that are arc-disjoint (we call such an orientation **good**). This is of course possible if and only if G contains a pair of edge-disjoint spanning trees such that the graph formed by the union of their edges has the desired orientation. Such a graph is called a 2T-graph. We show connections between the problem of deciding whether a given 2T-graph has a good orientation and matroid theory, in particular the rigidity matroid of a graph.

In the second part of the talk we consider antistrong digraphs, that is, digraphs in which there is an antidirected trail from x to y, starting and ending with a forward arc for every ordered pair of distinct vertices x, y. Antistrong connectivity is closely related to matroids and we show how use this connection to test in polynomial time whether a given digraph is antistrong.

Finally we consider the problem of deciding whether a given graph has an antistrong orientation and give a complete characterization of when this is the case. Again matroids form an important tool.

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*joint work with Stephane Bessy, Jing Huang, Bill Jackson and Matthias Kriesell

Tuesday 11.00

Bounds for convolutional codes

Ángela Barbero, Universidad de Valladolid and Øyvind Ytrehus, Simula UiB

A *q*-ary error correcting convolutional code is determined by a linear encoder, by which an *n*-bit output vector is produced as a linear combination of the current *k*-bit input vector and one or more previous input vectors. Convolutional codes are attractive in many cases because they offer a better application-dependent tradeoff between block size *n*, code rate *k/n*, decoding complexity, and/or decoding delay than corresponding (*q*-ary) block codes with error correction performance.

As for block codes, it is desirable to find the "best" convolutional code, given *q,n*, and *k*. While the problem of deriving bounds for block codes is a mature area, less is known about finding tight lower and upper bounds for convolutional codes. We will describe upper and lower bounds for convolutional codes.

Wednesday 11.00

Squares of cyclic codes

Ignacio Cascudo Pueyo, Aalborg Unversity

The square of a linear code C is the code spanned over the same finite field by the set of componentwise products of every pair of (non-necessarily distinct) codewords from C. This notion is interesting because of certain applications in particular in the area of secure multiparty computation within cryptography, where it is interesting to find codes with large rate whose squares have large minimum distance. I will explain some of my recent results where I construct cyclic codes with such characteristics, which in turn motivates a problem on finding large sets of integers with certain properties and counting their cardinalities.

Tuesday 14.00

LATIN SQUARES AND LATIN CUBES WITH FORBIDDEN ENTRIES

Carl Johan Casselgren

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An $n \times n$ partial Latin square is α-*sparse* if every row and column contains at most αn non-empty cells and each of the symbols $1, \ldots, n$ occurs in at most αn cells. An $n \times n$ array A where each cell contains a subset of $\{1, \ldots, n\}$ is a $(\beta n, \beta n, \beta n)$-*array* if each symbol occurs at most βn times in every row and column, and every cell contains a set of size at most βn. Combining the notions of completing partial Latin squares and avoiding arrays, we prove that there are constants $\alpha, \beta > 0$ such that, for every positive integer n, if P is an α-dense $n \times n$ partial Latin square, A is an $n \times n$ $(\beta n, \beta n, \beta n)$-array, and no cell of P contains a symbol that appears in the corresponding cell of A, then there is a completion of P that *avoids* A; that is, there is a Latin square L that agrees with P on every non-empty cell of P, and, for each i, j satisfying $1 \leq i, j \leq n$, the symbol in position (i, j) in L does not appear in the corresponding cell of A. We also consider similar questions for Latin Cubes.

This is joint work with L. Andrén, K. Markström and L.A. Pham.

Monday 15.30

Construction of biangular line sets in low-dimensional Euclidian spaces

Mikhail Ganzhinov, Aalto University

A set of lines in Euclidean space is called biangular if the angle between each pair of lines can have only two values. In this computer-aided work we use combined approach of isomorph-free exhaustive generation of candidate Gram matrices with indeterminate angles and Gröbner basis computation to classify the largest biangular sets of lines. Classification up to dimension 6 is obtained. This is joined work with Ferenc Szöllősi.

Tuesday 11.30

Symmetric Mosaics

Oliver W. Gnilke, Aalborg University

joint work with Padraig O'Cathain and Oktay Olmez

It is a classical result that given a matrix representation M of a design, its complement M' describes a design as well. Hence, they form a partition of the all ones matrix into two designs. Mosaics can be seen as the generalization of this statement, where we ask whether the all ones matrix can be partitioned into three or more designs. Some constructions of mosaics from resolvable designs have been described, but no non-trivial criterium for their existence has been established. We will investigate the special case of symmetric designs and show a Bruck-Ryser-Chowla type of criterion which helps to exclude many parameter sets. Furthermore, open questions regarding mosaics are presented.

Tuesday 14.30

A localization method in Hamiltonian graph theory

Jonas Granholm, Linköping University

Many classic criteria for the existence of Hamiltonian cycles in graphs relate vertex degrees to the number of vertices in the entire graph. Perhaps the most famous such result is the one by Dirac, which states that a graph G is Hamiltonian if every vertex of G is adjacent to at least half of all vertices of G. The classes of graphs covered by such theorems are necessarily limited to dense graphs of small diameter.

Beginning in the 1980's, Asratian and Khachatryan pioneered a method to overcome this limitation, by instead considering local structures of graphs. They obtained several generalizations of well-known sufficient conditions for Hamiltonicity, for instance the following generalization of Dirac's theorem: A graph G is Hamiltonian if every vertex u of G is adjacent to at least half of all vertices at distance at most 3 from u. In this talk we will discuss this localization method, and some recent results in the area.

Joint work with Armen S. Asratian and Nikolay K. Khachatryan

Tuesday 10.30

Improved Bounds on the Threshold Gap in Ramp Secret Sharing

Jaron Skovsted Gundersen, Aalborg University

Linear secret sharing schemes over finite fields are widely used in several cryptographic areas. Secret sharing deals with the case that a dealer distributes a secret among some participants in such a way that individual participants gain no information about the secret but uniting the shares from several participants make the participants able to recover the secret. We consider ramp secret sharing schemes meaning that the secret consists of several field elements while the shares distributed to the participants are just a single field element. We obtain lower bounds on the so-called threshold gap of such schemes, a measurement of how good secret sharing schemes are due to privacy and reconstruction. Our main result establishes a family of bounds on the threshold gap which are tighter than previously known bounds for linear ramp secret sharing schemes.

Tuesday 16.00

Degree Tables

Daniel Heinlein, Aalto University

Degree tables [1,2] are a combinatorial structure related to sumsets that arise in the context of secure distributed matrix multiplication [3] in which a user wishes to compute the product of two matrices with the assistance of honest but curious servers. The main questions about degree tables are lower and upper bounds on the number of entries and, analogously to sumsets, an inverse type problem to classify degree tables with a small number of entries. In this talk, we present new lower bounds, characteristics of some bound-achieving degree tables and the best known construction.

[1] R.G.L. D'Oliveira, S. El Rouayheb, and D. Karpuk, "GASP Codes for Secure Distributed Matrix Multiplication," arXiv:1812.09962, 2018.
[2] R.G.L. D'Oliveira, S. El Rouayheb, D. Heinlein, and D. Karpuk, "Degree Tables for Secure Distributed Matrix Multiplication," IEEE Information Theory Workshop, 2019.
[3] W.-T. Chang, R. Tandon, "On the Capacity of Secure Distributed Matrix Multiplication," arXiv:1806.00469, 2018.

Tuesday 15.30

Tropical homotopy continuation

Anders Nedergaard Jensen, Aarhus University

A tropical hypersurface is a polyhedral object that arises as the zero set of a multivariate polynomial when replacing addition and multiplication by maximum and addition and requiring the maximum to be attained at least twice. We present a "tropical" version of the numerical homotopy continuation algorithm that allows finding the isolated points in the intersection of n tropical hypersurfaces in n-dimensional space. Combined with the idea of regeneration from Numerical Algebraic Geometry we obtain an exact algorithm for finding the generic intersection and the associated mixed volume with a complexity similar to that of an algorithm by Malajovich.

Wednesday 10.30

Generalized weights of rank metric codes, a combinatorial approach

Trygve Johnsen, UiT - Arctic University of Norway

We sketch a way, in which one can define generalized weights of rank metric codes, in a way analogous to generalized Hamming weights for linear block codes.
There are several ways to do this in the existing litterature. In addition we show how one can describe an analogous way to describe generalized weights for objects called (q,m)-polymatroids. This definition corresponds in a natural way to the one we give for rank metric codes, and is used to show a form of Wei duality.
This is joint work with Sudhir Ghorpade.

Wednesday 11.30

Automorphisms of an srg$(162, 21, 0, 3)$

Leif Jørgensen
Aalborg University, Denmark

Maknev and Nosov [1] proved that if g is an automorphism of a strongly regular graph with parameters $(162, 21, 0, 3)$ then the subgraph fixed by g is either empty or $K_{1,3}$. And if there are no fixed vertices then an srg(81,20,1,6) can be obtained by collapsing the orbits. We present a computation showing that an automorphism of order 2 can not be without fixed vertices, and we show that an srg$(162, 21, 0, 3)$ is not vertex-transitive.

References

[1] A. A. Makhnev and V. V. Nosov, On automorphisms of strongly regular graphs with $\lambda = 0$ and $\mu = 3$ (Russian) *Algebra i Analiz 21* (2009), no. 5, 138–154; translation in St. Petersburg Math. J. 21 (2010), no. 5, 779–790.

Monday 16.30

On sequential basis replacement in matroids

Dani Kotlar, Elad Roda and Ran Ziv

Given a matroid $\mathcal{M}$, and two bases A and B in $\mathcal{M}$, the term *basis exchange* refers to exchanging elements between the two bases, so that the resulting sets are also bases. The term *basis replacement* refers to replacing elements in a given basis so that the resulting set is also a basis. The motivation for the current work is the following problem posed by Gabow in 1976 [3]:

Problem. *Given two bases A and B of a matroid $\mathcal{M}$, can we exchange the elements of A and B one by one, so that after each exchange the two resulting sets are bases?*

Partial results include solutions for graphic matroids [2], for sparse paving matroids [1] and for all matroids of ranks 4 and 5 [6, 4]. In this work we consider related problems on base replacement. We show [5] that given two bases A and B of a matroid $\mathcal{M}$, where the elements of B are given in a fixed order $b_1, \dots, b_n$, there is a bijection σ from B to A such that for each $i = 1, \dots, n$, both $A \setminus \{\sigma(b_i)\} \cup \{b_i\}$ and $A \setminus \{\sigma(b_1), \dots \sigma(b_i)\} \cup \{b_1, \dots, b_i\}$ are bases.

We give a proof for representable matroids relying on a formula of Dodgson, and a proof for general matroids applying basic properties of matroids.

References

[1] J. E. Bonin. Basis-exchange properties of sparse paving matroids. *Advances in Applied Mathematics*, 50(1):6–15, 2013.

[2] M. Farber, B. Richter, and H. Shank. Edge - disjoint spanning trees; a connectedness theorem. *Journal of Graph Theory*, 10:319–324, 1985.

[3] H. Gabow. Decomposing symmetric exchanges in matroid bases. *Mathematical Programming*, 10:271–276, 1976.

[4] D. Kotlar. On circuits and serial symmetric basis-exchange in matroids. *SIAM Journal on Discrete Mathematics*, 27(3):1274–1286, 2013.

[5] D. Kotlar, E. Roda, and R. Ziv. On sequential basis replacement in matroids. *a manuscript*, 2019.

[6] D. Kotlar and R. Ziv. On serial symmetric exchanges of matroid bases. *Journal of Graph Theory*, 73(3):296–304, 2013.

Monday 16.00

Quaternary complex Hadamard matrices of order 18

William T. Paavola
Aalto University, Finland

(Joint work with Patric R. J. Östergård)

Abstract

A *quaternary complex Hadamard matrix* H of order n is an $n \times n$ matrix over the complex 4th roots of unity such that $HH^* = nI_n$. We present the methods and results of a computer-aided classification of the quaternary complex Hadamard matrices of order 18. There are exactly 3 830 723 such matrices, up to monomial equivalence.

Tuesday 17.00

Real zeros and the alternatingly increasing property in algebraic combinatorics

Liam Solus, KTH Royal Institute of Technology

A central theme in algebraic, geometric, and topological combinatorics is the investigation of distributional properties of combinatorial generating polynomials such as symmetry, log-concavity, and unimodality. Recently, new questions in the field ask when such polynomials possess another distributional property, called the alternatingly increasing property, which implies unimodality. The alternatingly increasing property for a given polynomial is equivalent to a unique pair of symmetric polynomials both being unimodal with nonnegative coefficients. We will discuss a systematic approach to proving the alternatingly increasing property holds using real zeros of these symmetric polynomials. We will then look at some applications of these methods to recent questions and conjectures in algebraic combinatorics. This talk is based on joint work with Petter Brändén.

Tuesday 16.30

Hex: the full story

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Lektor emeritus

University of Southern Denmark, DK-5230 Odense M.

The boardgame Hex was discovered in the early 1940s in Denmark by Piet Hein. Newly found documents in the Jens Lindhard Archive of Aarhus University reveal how Piet Hein was able to publish and maintain a regular biweekly newspaper column of Hex, of his plans to publish a Hex book, and of his narrow escape in 1943. The full story of Hex is the subject of a recent book ***HEX: THE FULL STORY*** (published in February 2019 by CRC Press, Taylor&Francis, by Ryan Hayward with Bjarne Toft).

The talk will focus on:

1. 1943 and the happenings then.
2. Hex problems, making Hex a one-player game, rather than for two players.
3. Unsolved mathematical problems related to Hex.

The book has a wider scope, as shown in its summary:

Hex is the subject of books by Martin Gardner and Cameron Browne. Hex theory touches on graph theory, game theory and combinatorial game theory, with elegant proofs that the game has no draws and that the first player can win. From machines built by Claude Shannon to agents using Monte Carlo Tree Search, Hex is often used in the study of artificial intelligence. Written for a wide audience, this is the full story of Hex, inside and out, with all its twists and turns: Hein's creation, Lindhard's puzzles, Nash's proofs, Gale's Bridg-it, the game of Rex, Shannon's machines, Bridg-it's fall, Hex's resilience, Hex theory, the hunt for winning strategies, and the rise of Hexbots.

Tuesday 13.30

Title: Spanning eulerian subdigraphs avoiding k prescribed arcs in tournaments

Speaker: Anders Yeo

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Joint work with: Jørgen Bang-Jensen and Hugues Déprés

Abstract: A digraph is eulerian if it is connected and every vertex has its in-degree equal to its out-degree. Having a spanning eulerian subdigraph is thus a weakening of having a hamiltonian cycle. A digraph is semicomplete if it has no pair of non-adjacent vertices. A tournament is a semicomplete digraph without directed cycles of length 2. Fraise and Thomassen proved that every $(k+1)$-strong tournament has a hamiltonian cycle which avoids any prescribed set of k arcs. Bang-Jensen, Havet and Yeo recently demonstrated that a number of results concerning vertex-connectivity and hamiltonian cycles in tournaments have analogues when we replace vertex connectivity by arc-connectivity and hamiltonian cycles by spanning eulerian subdigraphs. They showed the existence of a smallest function $f(k)$ such that every $f(k)$-arc-strong semicomplete digraph has a spanning eulerian subdigraph which avoids any prescribed set of k arcs. They proved that $f(k) \leq \frac{(k+1)^2}{4} + 1$ and also proved that $f(k) = k + 1$ when $k = 2, 3$. Based on this they conjectured that $f(k) = k + 1$ for all $k \geq 0$.

We will show that f is in fact linear in k and give a proof of $f(k) \leq (\lceil \frac{3k+1}{2} \rceil)$. Thereafter we will indicate how this can be improved to $f(k) \leq (\lceil \frac{6k+1}{5} \rceil)$.

Monday 15.00

r-sparse Steiner Triple Systems

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Abstract

A (k, l)-configuration in a Steiner triple system STS(v) is a set of l blocks on k points. For Steiner triple systems, $(l+2, l)$-configurations are of particular interest. The smallest nontrivial such configurations include the Pasch configuration, which is a $(6, 4)$-configuration, and the mitre configuration, which is a $(7, 5)$-configuration. An STS(v) is r-sparse if it does not contain any $(l+2, l)$-configuration for $4 \leq l \leq r$. The existence problem for anti-Pasch and anti-mitre Steiner triple systems has been solved, but such objects have been classified only up to order 19. In the current work, a computer-aided classification shows that there are 83003869 isomorphism classes of anti-Pasch STS(21). Exploration of the classified systems reveals that there are three 5-sparse STS(21) but no 6-sparse STS(21). The anti-Pasch STS(21) further lead to 14 Kirkman triple systems, none of which is doubly resolvable. This is joint work with Janne Kokkala.

Monday 17.00